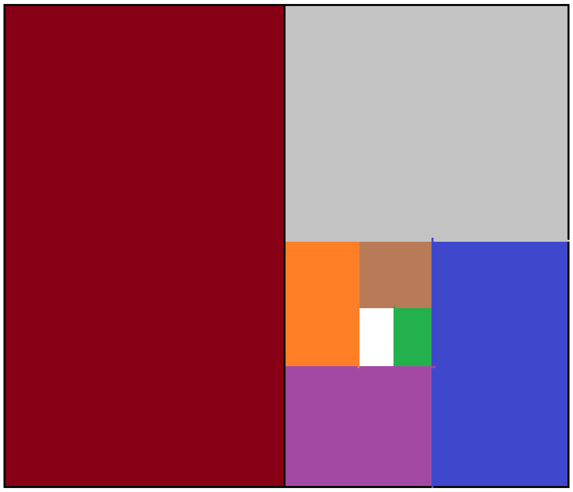
Algebra II

Sequence & Series Packet



The geometric ratio is 1/2...

The first term is 1/2

$$S_{\infty} = \frac{1/2}{1 - 1/2} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \dots = 1$$

Contents include notes, examples, and practice test (& solutions)

Sequences and Series

Sequence

What is it? A list of numbers (set apart by commas) that often have a pattern.

The following are common examples:

Arithmetic: Each term is separated by a "common difference"

Example: 2, 5, 8, 11, 14, ... common difference: 3

13, 9, 5, 1, -3, -7, ... common difference: -4

Geometric: Each term is separated by the same factor or "common ratio"

Example: 3, 6, 12, 24, 48, 96, ... common ratio: 2

16, 8, 4, 2, 1, 1/2, 1/4, ... common ratio: 1/2

And special sequences,

Fibonacci: Each term is the sum of the previous 2 terms

Example: 1, 1, 2, 3, 5, 8, 13, 21, ...

Square Numbers: Each term is the "square of that position"

Example: 1, 4, 9, 16, 25, 36, 49, ...

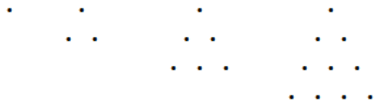
1^2 2^2 3^2 4^2 etc...

Cube Numbers: Each term is the "cube of that position"

Example: 1, 8, 27, 64, 125, ...

1^3 2^3 3^3 4^3 etc...

Triangle Numbers: 'Dots in a triangle' -- 1, 3, 6, 10, 15, 21, 28



(adding a row each time)

Identify the pattern and find the missing term.

-7, ____, 7, 14, 21, ...

2, -4, 8, ____, 32, ...

4, 7, 12, 19, ____, 39, 52, ...

4, 7, 11, 18, 29, ____, 76, ...

(Answers on the next page)

Identify the pattern and find the missing term.

-7, 0, 7, 14, 21, ...	Arithmetic sequence: common difference is 7 (Add 7, add 7, add 7, ...)
2, -4, 8, -16, 32, ...	Geometric sequence: common ratio is -2 (multiply by -2 each time)
4, 7, 12, 19, 28, 39, 52, ...	Square number sequence (plus 3) (or, add 3, add 5, add 7, add 9, etc...)
4, 7, 11, 18, 29, 47, 76, ...	'Fibonacci' sequence (4 + 7 = 11 7 + 11 = 18 11 + 18 = 29 etc...)

Formula to find the nth term in an arithmetic sequence

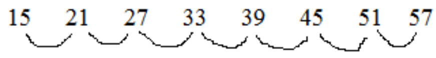
$$a_n = a_1 + (n - 1)d$$

a_1 is the first term of the sequence

d is the common difference

example: first term is 15; common difference is 6... Find the 8th term.

$$t_8 = 15 + (8 - 1)6 = 57$$



There are 7 moves from the 1st term to the 8th

Each move is 6 spaces...

7 x 6 = 42 spaces from the starting point...

15 + 42 = 57

example: (Arithmetic sequence) fourth term is 38; common difference is 5.. Find the 12th term..

In this case we are given the 4th term (instead of the 1st term).. So, we will solve intuitively..

How many moves from the 4th term to the 12th term? 8 moves
How many spaces is each move? 5 (the common difference)
Total spaces from the starting point? 5 x 8 = 40

Therefore, the 12th term is 38 + 40 = 78

Exercises:

1) The arithmetic sequence is 28, 25, 22, ...

   a) Find the 5th term

   b) Find terms 200 and 201

   c) 1 is a term in this sequence. Which is it?

2) In an arithmetic sequence, the 7th term is 32 and the common difference is -5.

   a) What is the first term in the sequence?

   b) Is -23 a term in this sequence? If so, which term?

(answers on following page)

Exercises:

1) The arithmetic sequence is 28, 25, 22, ...

a) Find the 5th term

b) Find terms 200 and 201

c) 1 is a term in this sequence. Which is it?

2) In an arithmetic sequence, the 7th term is 32 and the common difference is -5.

a) What is the first term in the sequence?

b) Is -23 a term in this sequence? If so, which term?

$a_n = a_1 + (n-1)d$

1) 28, 25, 22...

a) 28, 25, 22, 19, $\boxed{16,}$ ...

b) Using the formula: $a_{200} = 28 + (200-1)(-3)$

$= 28 + (199)(-3) = \boxed{-569}$

(And, the $\boxed{\text{201st term is } -572}$)

c) $1 = 28 + (n-1)(-3)$

$-27 = (n-1)(-3)$

$9 = n - 1$

$\boxed{n = 10}$

2) a) $a_7 = a_1 + (7-1)(-5)$

$32 = a_1 + (-30)$

$\boxed{62 = a_1}$

62, 57, 52, 47, 42, 37, 32, ...

b) 62 − (-23) = 85.. Since 85 is a multiple of -5, it is in the sequence.

$-23 = 62 + (n-1)(-5) \longrightarrow \boxed{n = 18}$

Formula to find the n^{th} term in a geometric sequence:

$$a_n = a_1 \cdot r^{n-1}$$

a_1 is the first term
r is the common ratio

Example:

The geometric sequence has a first term of 2000; common ratio: .8

Find the 2nd and 3rd terms: 2000, (2000 x .8), (2000 x .8)(.8), ((2000 x .8)(.8))(.8), etc..
2000, $\boxed{1600, \quad 1280,}$ 1024, etc...

Find the 25th term: (using the formula) $a_{25} = 2000 \cdot (.8)^{24} = \boxed{9.444733}$

268.435 is a term in the sequence. Which is it? $268.435 = 2000 \cdot (.8)^{n-1}$

$.13422 = (.8)^{n-1}$ (using logarithms) $\boxed{n = 10}$

Series

What is it? The *sum of the terms* of a sequence.

$S_7 = 6 + 3 + 0 + (-3) + (-6) + (-9) + (-12) = -21$

$S_5 = 3 + 8 + 13 + 18 + 23 = 65$

Arithmetic Series Formula:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Example: Find the sum of the first 20 terms of the following arithmetic series:
$a_n = 4 + 6n$

First term: $a_1 = 10$
20th term: $a_{20} = 124$

$S_{20} = \frac{20(10 + 124)}{2} = 1340$

Examine the patterns:

10 + 16 + 22 + 28 .... + 112 + 118 + 124

1st term + 20th term = 10 + 124 = 134
2nd term + 19th term = 16 + 118 = 134
3rd term + 18th term = 22 + 112 = 134
So, each pair is 134...
How many pairs? 20/2 = 10 pairs

10 pairs x 134/pair = 1340

Geometric Series Formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Example: Find the 8th partial sum of a geometric series of first term 3 and common ratio 2.

The sum will be $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384$

(The first term is 3; every term is 2x the previous)

Using the formula on the left, the 8th partial sum is

$$S_8 = \frac{3(1 - 2^8)}{1 - 2} = \frac{3(1 - 256)}{-1} = 765$$

Infinite Series

What is it? A series that has no last term. Since it has no ending, there is no definite value for an infinite series. However, there may be a limit.

Convergence and Divergence:

If the series has a limit, then it converges. If the series has no limit, then it diverges.

All arithmetic series diverge.

Example: $2 + 5 + 8 + 11 + \ldots$ goes to infinity... $S_\infty = \infty$

$-1/2 + (-1) + (-3/2) + (-2) + \ldots$ goes to negative infinity... $S_\infty = -\infty$

Some geometric series converge. If r is the common ratio, and if

$|r| < 1$, then the geometric series converges (to a limit)

Example: $1 + 3 + 9 + 27 + \ldots$ common ratio: 3 (greater than 1; diverges)

$2 + (-4) + 8 + (-16) + 32 + (-64) + \ldots$ common ratio: -2 (since $|-2| > 1$, the series diverges)

(note: if you pair the numbers, the series is $2 + 4 + 16 + 64 + \ldots$ and obviously diverges)

$$2 + \underbrace{(-4) + 8}_{4} + \underbrace{(-16) + 32}_{16} + \underbrace{(-64) + \ldots}_{64}$$

Infinite Geometric Series

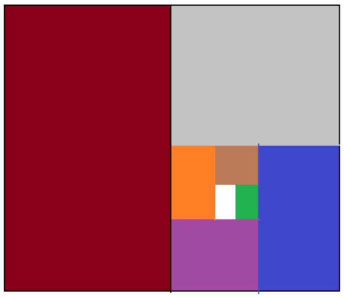
$$S_\infty = \frac{a_1}{1 - r}$$

$3 + 1 + 1/3 + 1/9 + 1/27 + \ldots$ common ratio: 1/3 (less than 1; converges)

Using the formula, $S_\infty = \frac{3}{(1 - 1/3)} = \frac{9}{2} = 4\frac{1}{2}$

(note: $3 + 1 + 1/3 + 1/9 + \ldots$ will end at some point, because (1/3)n approaches 0 as n gets larger and larger.. Therefore, the series *will approach* a specific value as the terms get closer to 0.)

Geometric series illustration:



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \ldots = 1$$

The geometric ratio is 1/2...

The first term is 1/2

$$S_\infty = \frac{1/2}{1 - 1/2} = 1$$

** Note: the multi-color box illustrates the infinite series is *approaching* 1

More topics, applications, and examples of Sequences and Series:

I. Using Geometric Series to express repeating decimals..

Convert $.21\overline{21}$ to a rational expression.

1) fraction method $.21\overline{21}$ ----> 21/99

Let $n = .21\overline{21}$

Then $100n = 21.21\overline{21}$

$$\begin{array}{rr} 100n & 21.21\overline{21} \\ -\quad n & -\ .21\overline{21} \\ \hline 99n & 21 \end{array}$$

$99n = 21$

So, $n = \frac{21}{99}$.

2) geometric series .21 + .0021 + .000021 + ... ---> $S_n = .21(.01)^{n-1}$

geometric ratio: .01

$$S_\infty = \frac{.21}{(1-.01)} = \frac{.21}{.99} = \frac{21}{99}$$

II. Applications of geometric sequences and series (word problem):

A tree grows 40 inches its first year. It grows 38 inches the second year. Assuming a geometric rate of growth,

a) how much will it grow the 5th year?

b) How tall will it be after 10 years?

c) What will its ultimate height be?

find geometric ratio: 38/40 = .95

$T_1 = 40$

$T_2 = 38$

$T_5 = 40(.95)^4 = 32.58$

$$S_{10} = \frac{40(1-.95^{10})}{(1-.95)} = \frac{.401}{.05}$$ 321 inches

Ultimate height: $\frac{40}{(1-.95)} =$ 800 inches

III. Explicit vs. Recursive formulas

Explicit formulas use direct calculation and simply require the term..

Recursive formulas require the 1st term of the sequence and the computation of previous terms.

Example: 5, 10, 15, 20, 25, ...

Explicit formula: $a_n = 5n$

Recursive formula: $a_1 = 5$

$a_n = a_{n-1} + 5$

Using the explicit form, you find the 10th term by calculating 5(10)

Using the recursive formula, the 10th term is found by adding 5 to the 9th term..

Example: 4, 12, 36, 108, ...

Explicit formula: $a_n = 4 \cdot 3^{n-1}$

Recursive formula: $a_1 = 4$

$a_n = 3 \cdot a_{n-1}$

Each term is 3 times the previous the term.

Recursive formulas are useful in 'Fibonacci' type sequences..

Example: 2, 3, 5, 8, 13, 21, ...

$a_1 = 2$

$a_2 = 3$

$a_n = a_{n-1} + a_{n-2}$

There is no explicit formula to express this sequence pattern.

IV. Finding the "interval of convergence" of a geometric series.

Example: Determine the interval of convergence of the following series.

$$S_n = \left(\frac{x+2}{5}\right)^n$$

What values of x will make this series converge?
Since $-1 < r < 1$, find the range of x values that will keep the common ratio less than 1.

$$-1 < \left(\frac{x+2}{5}\right) < 1$$

$$-7 < x < 3$$

(-7, 3) is the interval of convergence.

If x is between -7 and 3, then the geometric series will converge to a specific value.

V. Sequence vs. Series

Example Write the 1st 4 terms of the following:

a) arithmetic sequence w/1st term 10 and common difference 2 10, 12, 14, 16 (terms separated by commas)

b) geometric series w/1st term 3 and common ratio 3 $3 + 9 + 27 + 81$ (terms added together)

VI. Arithmetic and Geometric Means

Arithmetic mean:

What is it? The average of two numbers; The middle of terms in a sequence.

To find the arithmetic mean of 2 terms, add them and divide by 2.

Arithmetic mean of 10 and 16: 13
(10 + 16)/2 = 13
10, 13, 16 common difference: 3

Geometric mean:

What is it? A type of average; The middle of terms in a geometric sequence.

To find the geometric mean of 2 terms, multiply them and take the square root.

Geometric mean of 2 and 8: $\sqrt{16} = 4$
2 x 2 = 4 4 x 2 = 8
2 , 4, 8 common ratio: 2

Example: $T_3 = 4$ $T_5 = 16$ What is the mean (T_4)? The *arithmetic* mean is 10.
The *geometric* mean is 8.

Note: You must specify the type of sequence!

"multiple means"

Insert 3 geometric means between 2 and 32.. includes imaginary numbers!

$2, \boxed{4, 8, 16,} 32$
$2, \boxed{-4, 8, -16,} 32$
$2, \boxed{4i, -8, 16i,} 32$
$2, \boxed{-4i, 8, -16i,} 32$

Insert 2 arithmetic means between 7 and 22

$$\frac{22 - 7}{3} = 5$$

7, (7 + 5), (7 + 5 + 5), 22
$7, \boxed{12, 17,} 22$

VII. Summation and sigma notation:

$$\sum cx = c\sum x \qquad \sum_{i=1}^{n} c = nc \qquad \sum x + y = \sum x + \sum y \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Example: Find $\sum_{n=1}^{10} 3n + 6$

$$\sum_{n=1}^{10} 3n + \sum_{n=1}^{10} 6$$

$$3\sum_{n=1}^{10} n = 3\left(\frac{10(11)}{2}\right) = 165$$

$$\sum_{n=1}^{10} 6 = 10(6) = 60$$

$$165 + 60 = \boxed{225}$$

SEQUENCE AND SERIES

PRACTICE TEST

Sequences & Series Test

I. Sequence Patterns

A. Add 2 terms to each of the following:

1) 6, 12, 20, 30, 42, 56,

2) 3, −6, 12, −24, 48,

3) $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$,

4) 2, 3, 5, 8, 13, 21,

B. Identify the sequences (geometric, arithmetic, or neither); Give the common ratio or difference, (if one exists).

1) 3, −9, 27, −81, …

sequence type:
common ratio/difference:

2) 2, 7, 9, 16, 25, …

sequence type:
common ratio/difference:

3) $\frac{1}{3}$, $\frac{5}{6}$, $\frac{4}{3}$, 1 5/6 …

sequence type:
common ratio/difference:

II. Sequence Expressions

A. Express the following in *Explicit form* and *Recursive form.*

1) 3, 10, 17, 24, …
(arithmetic)

Explicit Form:

Recursive Form:

2) 32, 16, 8, 4, 2, …
(geometric)

Explicit Form:

Recursive Form:

B) Answer the questions; then, write expressions to describe each sequence.

1) Arithmetic; $T_4 = 28$ $T_{10} = 58$

What is T_1 ?

T_{20}?

What is the common difference of the sequence?

$T_n =$

2) Geometric; $T_3 = \frac{1}{8}$ $T_6 = \frac{1}{64}$

What is T_1 ?

What is the common ratio of the sequence?

$T_n =$

III. Series

A. Solve the following

1) Arithmetic; 1st term: 7 common difference: 4

$S_6 =$

2) Geometric; 1st term: 5 common ratio: 2

$S_4 =$

B. Answer the following (using formulas)

1) Find the 10th partial sum of the geometric series with 1st term 400 and common ratio .9

2) Find the 40th partial sum of the following:

$32 + 38 + 44 + \ldots$

IV. Summations and Sigma Notation

A) Solve

1) $\sum_{T=1}^{5} T^2 =$

2) $\sum_{X=1}^{20} 3X + 5 =$

3) $\sum_{m=3}^{7} 4m - 3 =$

B) Describe the following series using sigma notation

1) $4 + 8 + 12 + 16 =$

$\sum$

2) $2 + 6 + 18 + 54 + 162 =$

$\sum$

V. Arithmetic and Geometric means

A) Find the following

1) 3 and 9 arithmetic mean:

2) 3 and 48 all possible geometric means:

VI. Geometric Series: convergence/divergence

A) Determine whether the following geometric series converge or diverge.
then, find the limit of convergence (if it exists).

1) 30 + 27 + ...

2) 10 + 13 + ...

B) Write $.27\overline{27}$ as an infinite geometric series; Then, express $.27\overline{27}$ as a fraction.

C) What is the interval of convergence for the following geometric series ?

$$\sum_{n=1}^{\infty} = \left(\frac{X-2}{3}\right)^n$$

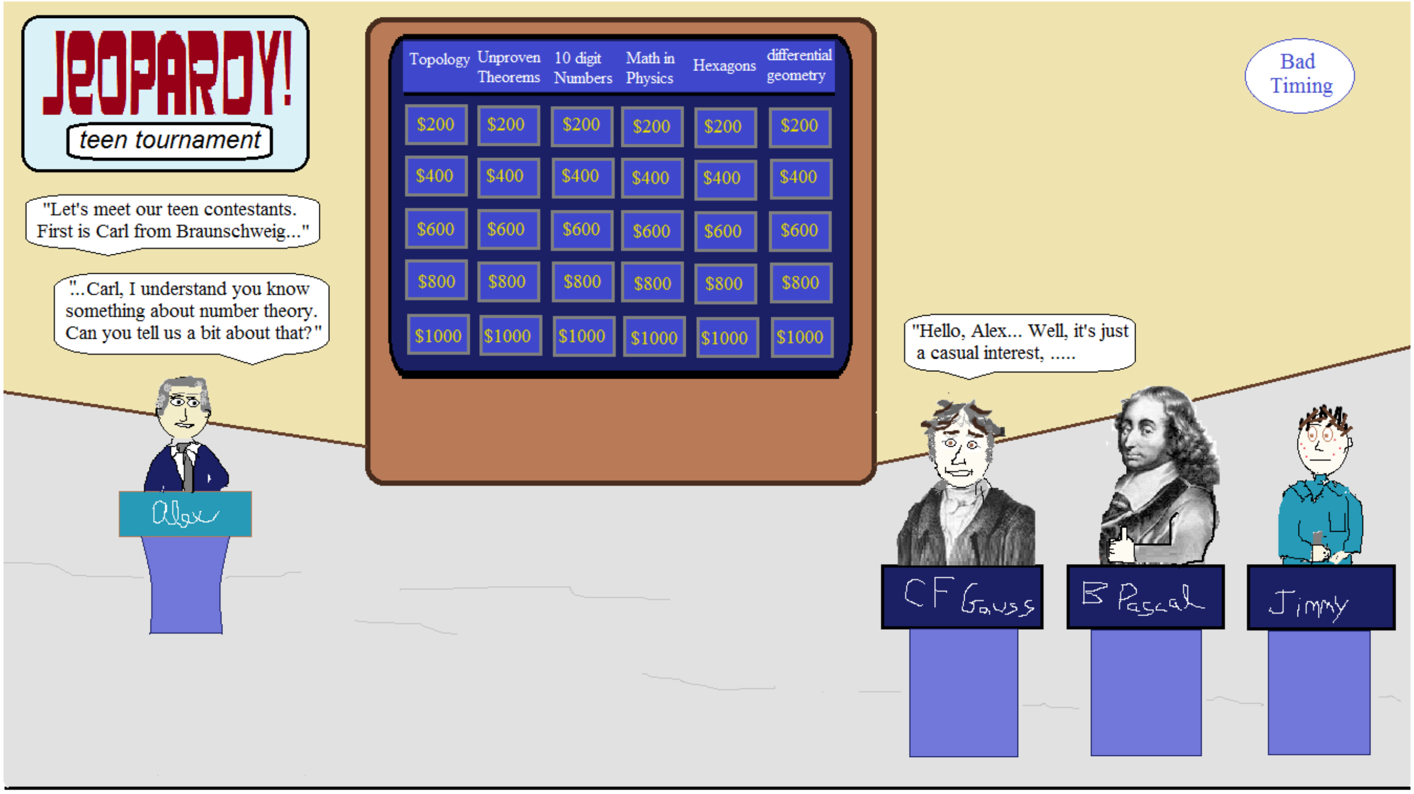
D) Answer the following:

1) Evaluate S_{∞} for 1/2, 1/4, 1/8, 1/16, ...

2) $\sum_{n=1}^{\infty} (1.001)^n =$

3) $\sum_{n=1}^{\infty} 3 - \left(\frac{2}{3}\right)^n =$

4) $\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n =$



Jimmy didn't have a chance...

L. Friedman #46 8-17-12
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SEQUENCE AND SERIES

ANSWERS

Sequences & Series Test

SOLUTIONS

I. Sequence Patterns

A. Add 2 terms to each of the following:

1) 6, 12, 20, 30, 42, 56, 72, 90 (add 6, add 8, add 10, add 12, etc...)

2) 3, −6, 12, −24, 48, −96, 192 (multiply by -2)

3) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}$ (add 2/4)

4) 2, 3, 5, 8, 13, 21, 34, 55 (Fibonacci sequence: each term is the sum of the previous two terms)

B. Identify the sequences (geometric, arithmetic, or neither); Give the common ratio or difference, (if one exists).

1) 3, −9, 27, −81, ...
sequence type: geometric
common ratio/difference: −3

2) 2, 7, 9, 16, 25, ...
sequence type: neither (each element is the sum of the previous two)
common ratio/difference: none

3) $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}$, 1 5/6 ...
sequence type: arithmetic
common ratio/difference: $\frac{1}{2}$

II. Sequence Expressions

A. Express the following in *Explicit form* and *Recursive form.*

1) 3, 10, 17, 24, ...
(arithmetic)

Explicit Form:

$A_n = 3 + 7(n - 1)$

Recursive Form:

$A_1 = 3$

$A_n = A_{n-1} + 7$

2) 32, 16, 8, 4, 2, ...
(geometric)

Explicit Form:

$G_n = 32 \left(\frac{1}{2}\right)^{n-1}$

Recursive Form:

$G_1 = 32$

$G_n = G_{n-1}\left(\frac{1}{2}\right)$

B) Answer the questions; then, write expressions to describe each sequence.

1) Arithmetic; $T_4 = 28$ $T_{10} = 58$

$\frac{58 - 28}{(10 - 4)} = 5$

What is T_1 ? $T_1 = T_4 - 5(3)$ $\boxed{T_1 = 13}$

T_{20}? $T_{20} = T_{10} + 5(10)$ $\boxed{T_{20} = 108}$

What is the common difference of the sequence?

common difference: 5

$T_n = 13 + 5(n - 1)$

2) Geometric; $T_3 = \frac{1}{8}$ $T_6 = \frac{1}{64}$

$\frac{1}{64} \div \frac{1}{8} = \frac{1}{8}$

(since T_3 and T_6 are 3 terms apart, we take the cube root of their quotient)

$\sqrt[3]{\frac{1}{8}} = 1/2$

What is T_1 ? $T_3 = (T_1)^3$

$\frac{1}{8} = \boxed{\left(\frac{1}{2}\right)}^3$

What is the common ratio of the sequence?

common ratio: $\frac{1}{2}$

$T_n = \left(\frac{1}{2}\right)^n$

Sequences & Series Test

SOLUTIONS

III. Series

A. Solve the following

1) Arithmetic; 1st term: 7 common difference: 4

$\boxed{S_6 = 102}$

34

$7 + 11 + 15 + 19 + 23 + 27$

34

34

– each pair is 34

– there are 3 pairs

$$\boxed{S_n = \frac{n(a_1 + a_n)}{2}}$$ Arithmetic

2) Geometric; 1st term: 5 common ratio: 2

$\boxed{S_4 = 75}$

$5 + 10 + 20 + 40$

$$= \frac{5(1-2^4)}{1-2} = \frac{-75}{-1} = 75$$

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$ Geometric

B. Answer the following (using formulas)

1) Find the 10th partial sum of the geometric series with 1st term 400 and common ratio .9

$$\frac{a_1(1-r^n)}{1-r} = \frac{400\,(1-.9^{10})}{1-.9} = \frac{260.5}{.1} = \boxed{2605}$$

2) Find the 40th partial sum of the following:

$32 + 38 + 44 + \ldots$

First term: 32

Common difference: 6

40th term: $32 + 6(39) = 266$

$$\frac{n(a_1 + a_n)}{2} = \frac{40(32+266)}{2} = 20(298) = \boxed{5960}$$

IV. Summations and Sigma Notation

A) Solve

1) $\sum_{T=1}^{5} T^2 = 55$

$T_1 = 1$

$T_2 = 4$ $1 + 4 + 9 + 16 + 25 = 55$

$T_3 = 9$

$T_4 = 16$

$T_5 = 25$

2) $\sum_{X=1}^{20} 3X + 5 = 730$

$\sum_{X=1}^{20} 5 = 5(20) = 100$

$3\sum_{X=1}^{20} X = 3\left(\frac{20 \cdot 21}{2}\right) = 630$

3) $\sum_{m=3}^{7} 4m - 3 = 85$

$m_3 = 9$

$m_4 = 13$ $9 + 13 + 17 + 21 + 25 = 85$

$m_5 = 17$

$m_6 = 21$

$m_7 = 25$

B) Describe the following series using sigma notation

1) $4 + 8 + 12 + 16 =$

$$\sum_{i=1}^{4} 4i$$

2) $2 + 6 + 18 + 54 + 162 =$

$$\sum_{k=1}^{5} 2 \cdot 3^{(k-1)}$$

Sequences & Series Test

V. Arithmetic and Geometric means

A) Find the following

1) 3 and 9 arithmetic mean: $\boxed{6}$ (common difference is 3)

2) 3 and 48 all possible geometric means: $\boxed{\begin{matrix}12\\-12\end{matrix}}$ (common ratio: 4)
(common ratio: -4)

VI. Geometric Series: convergence/divergence

A) Determine whether the following geometric series converge or diverge.
then, find the limit of convergence (if it exists).

1) 30 + 27 + ...

$27 \div 30 = .9$

.9 is common ratio

Since .9 < 1, it does converge.

$$\frac{a}{1-r} = \frac{30}{1-.9}\boxed{= 300}$$

2) 10 + 13 + ...

The common ratio is

1.3 (10 x 1.3 = 13) Since 1.3 > 1, the series $\boxed{\text{diverges...}}$

B) Write $.27\overline{27}$ as an infinite geometric series; Then, express $.27\overline{27}$ as a fraction.

$.27\overline{27} = .27 + .0027 + .000027 + \ldots$

common ratio r = .01

$$\boxed{\sum_{n=1}^{\infty} .27(.01)^{n-1}} \qquad \frac{a}{1-r} = \frac{.27}{(1-.01)} = \frac{.27}{.99} = \boxed{\frac{27}{99}}$$

Also,
$n = .27\overline{27}$
$100n = 27.27\overline{27}$
$100n - n = 27.27\overline{27} - .27\overline{27}$
$99n = 27$
$n = \frac{27}{99}$

C) What is the interval of convergence for the following geometric series?

$$\sum_{n=1}^{\infty} = \left(\frac{X-2}{3}\right)^n \qquad \left|\frac{X-2}{3}\right| < 1 \qquad \frac{X-2}{3} < 1 \quad \text{and} \quad \frac{X-2}{3} > -1$$

$$X < 5 \qquad X > -1$$

The interval of convergence for the series is
$-1 < X < 5$

(For the geometric series to converge, it must be less than 1)

D) Answer the following:

1) Evaluate S_∞ for 1/2, 1/4, 1/8, 1/16, ...

common ratio = 1/2
Since it is less than 1, it converges..

$$\frac{1/2}{(1-1/2)} \boxed{= 1}$$

2) $\sum_{n=1}^{\infty} (1.001)^n = \boxed{\infty}$ (because 1.001 > 1)

3) $\sum_{n=1}^{\infty} 3 - \left(\frac{2}{3}\right)^n = \boxed{\infty}$ (although $\left(\frac{2}{3}\right)^n$ will converge, 3 does not..)

4) $\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n = 3\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = 3\,\frac{2/3}{(1-2/3)} = 3 \times 2 = \boxed{6}$

$$\sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n = 2 + 12/9 + 24/27 + 48/81 + \ldots.$$

(each term in the series is less than the previous term -- the series is converging)

Thank you for downloading this packet…

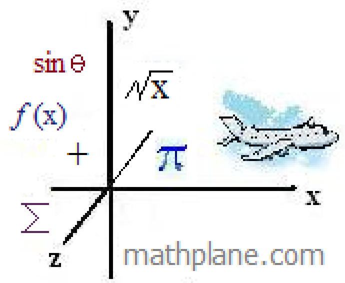
Hope the notes and practice test were a helpful review!

Questions, suggestions, or feedback is always appreciated…

Best always,
Lance

(If you found an error, let me know…)

Lance@mathplane.com



More stuff -→

"Multiple Arithmetic and Geometric Means"

Example: Find the *three arithmetic* means between 15 and 43?

The space between 15 and 43 is 28 units...

Since we're looking for 3 arithmetic means, there will be 4 (common) differences in the 28 units...

28/4 = 7 (common difference)...

15 22 29 36 43

Example: Find the *two geometric* means between 20 and 160?

Interpretation 1:

Find the geometric mean between 20 and 160

$160 \div 20 = 8 \longrightarrow \sqrt{8} = 2\sqrt{2}$

$2\sqrt{2}$ is the common ratio

20 $40\sqrt{2}$ 160

OR

$-2\sqrt{2}$ is the common ratio

20 $-40\sqrt{2}$ 160

Interpretation 2:

Insert 2 geometric means between 20 and 160

$160 \div 20 = 8 \longrightarrow \sqrt[3]{8} = 2$

2 is the common ratio

20 40 80 160

Example: Give 4 possible answers (including complex numbers):

three geometric means between 2 and 162.

2, 6, 18, 54, 162 common ratio: 3

2, -6, 18, -54, 162 common ratio: -3

2, $6i$, -18, $-54i$, 162 common ratio: $3i$

2, $-6i$, -18, $54i$, 162 common ratio: $-3i$

$$\frac{262}{2} = 81$$

$$\sqrt[4]{81} = 3, -3, 3i, -3i$$

Example: Evaluate $\sum_{k=1}^{10} (k+1)^2$

Method 1: Write out the terms (and seek pattern)

4 + 9 + 16 + 25 + 36 ...

(perfect squares)

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Method 2: Expand the term (and separate)

$$\sum_{k=1}^{10} k^2 + 2k + 1$$

$$\sum_{k=1}^{10} k^2 + 2\sum_{k=1}^{10} k + \sum_{k=1}^{10} 1$$

$$\frac{(10)(11)(21)}{6} + 2(55) + 10(1) = 505$$

Example: Write the following series in sigma notation: $-8 - 3 + 2 + 7 + \ldots + 62$

$$\sum_{k=0}^{n} a_1 + dk$$

a = 1st value
d = common difference
n = number of terms

This is an arithmetic series with common difference 5

(and, there are 15 terms)

$$\frac{62-(-8)}{5} = 14\ (+1) = 15$$

$$\sum_{k=0}^{14} -8 + 5k \quad \text{or} \quad \sum_{k=1}^{15} -8 + 5(k-1)$$

Example: Write the following series in sigma notation: $2 - 8 + 32 - 128 + \ldots$

This is a geometric series with common ratio (-4)

$$\sum_{k=1}^{\infty} 2 \cdot 4^{k-1} \quad \text{or} \quad 2^{2k-1}$$

Example: Evaluate $\sum_{n=0}^{20} \sqrt{2}\, n - 7$

$$\sqrt{2}\sum_{n=0}^{20} n - \sum_{n=0}^{20} 7$$

n = 0, 1st term = 0

$\underline{\text{21 terms}}$ x 7 = 147

$$\sqrt{2}\sum_{n=1}^{20} n = \sqrt{2}\,\frac{20(1+20)}{2} = 210\sqrt{2}$$

$$\boxed{\text{Total: } 210\sqrt{2} - 147}$$

Example: Find the common ratio of the sequence

$$5, 5^{7/6}, 5^{4/3}, 5^{3/2}, \ldots$$

simply divide any term by its previous term...

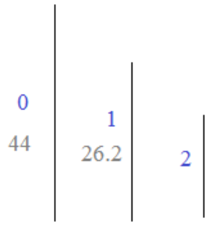
$$\frac{5^{7/6}}{5^{6/6}} = \boxed{5^{1/6}} \qquad \frac{5^{4/3}}{5^{7/6}} = \frac{5^{8/6}}{5^{7/6}} = \boxed{5^{1/6}}$$

Sequences and Series: Word Problems & Applications

Example: A rubber ball is dropped from 44 inches above the ground.
Each time it bounces, the ball retraces 60% of its previous height.

a) What is the height of the ball *after* the 8th bounce?

b) How far will the ball travel before it 'comes to rest'?



The height after a particular bounce can be expressed as

$a_0 = 44$

$a_1 = 44\,(.60) = 26.2$ inches

$$\boxed{a_n = 44(.60)^n}$$

$a_2 = 44\,(.60)(.60)$

$a_8 = 44(.60)^8 = \boxed{.739 \text{ cm above ground}}$

$$\boxed{S_\infty = \frac{a_1}{(1-r)}}$$

The distance travel will be the sum of all the bounces (up and down).
And, the number of bounces will be infinite...

distance traveled (going down): initial move: 44 cm..

$$S_{down} = \frac{44}{(1-\ .60)} = 110 \text{ cm}$$

distance traveled (going up): inital move: 26.2 cm..

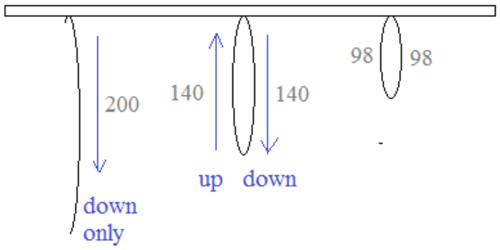
$$S_{up} = \frac{26.2}{(1\ -\ .60)} = 65.5 \text{ cm}$$

$$\boxed{\text{Total distance traveled will approach } 175.5 \text{ cm}}$$

Example: A bungee jumper leaps off a bridge and falls 200 feet before bouncing up 70%...
Then, the jumper falls again, before bouncing up 70%... This continues until the jumper settles.

a) After 5 bounces, how far has the bungee jumper traveled?

b) How far would the bungee jumper travel before stopping?



This is a geometric series: the first term is 200.
and, the common ratio is .70 or $\frac{7}{10}$

must add the downward sum AND the upward sum

$$\underset{down}{S_5} = \frac{200\,(1-.7^5)}{1-.7} = 554.62$$

$$\underset{up}{S_5} = \frac{140\,(1-.7^5)}{1-.7} = 388.23$$

$$\boxed{\begin{array}{c}\text{Total after 5 bounces:}\\ \text{942.85 feet}\end{array}}$$

$$\underset{down}{S_\infty} = \frac{200}{(1-.70)} = 666\frac{2}{3}$$

$$\underset{up}{S_\infty} = \frac{140}{(1-.70)} = 466\frac{2}{3}$$

Check:	down	up
	200	140
	140	98
	98	68.6
	68.6	48.02
	48.02	33.61
	554.62 ✓	388.23 ✓

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

Total distance traveled will go to $\boxed{1133\frac{1}{3} \text{ feet}}$

find the 10th term of $(a^5 - b^2)^{13}$

coefficient: $\frac{13!}{9!4!}$

$- \ (a^5)^4 (b^2)^9$

Find the sum of multiples of 6 *between 8 and 602*

30,294

first find number of terms..
and. find the last term...
then, find the partial sum where $t_1 = 12$ and $t_n = 600$

Find sum: $-6 + 3 - \frac{3}{2} + \frac{3}{4} - \ldots$

$$\boxed{S_\infty = \frac{a_1}{1 - r}}$$

Step 1: Determine the series

This is a geometric series

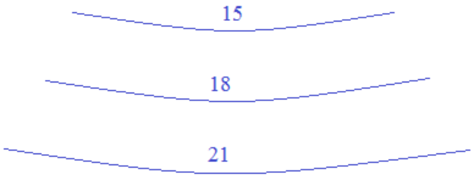
Step 2: Find the ratio

$\frac{a_{n+1}}{a_n} = \frac{-1}{2}$ i.e $\frac{3}{-6}$ $\frac{-3/2}{3}$ $\frac{3/4}{-3/2}$

common ratio is -1/2

Step 3: Use formula
(since common ratio < 1, this series will converge...)

An architect is designing a theater with 15 seats in the front row. Then, 18 seats in the second row.
Then, 21 seats in the third row. And, each row with 3 seats more than the previous row.
If the theater requires at least 900 seats, how many rows will the theater require?



This is an arithmetic series, because each term is 3 more than the previous..

Common difference: $d = 3$

First term: $a_1 = 15$

The nth term: $a_n = 3(n - 1) + 15 \ \text{--->} \ 3n + 12$

The sum of all the seats: $S_n = \frac{n(a_1 + a_n)}{2}$

$$900 = \frac{n(15 + 3n + 12)}{2}$$

$$1800 = 3n^2 + 27n$$

$$0 = 3n^2 + 27n - 1800$$

$$0 = 3(n^2 + 9n - 600)$$

Using quadratic formula, n = -29.4 or 20.4

Since the number of rows (n) cannot be
negative, the architect needs at least 20.4 rows..
So, the theater will have at least 21 rows...

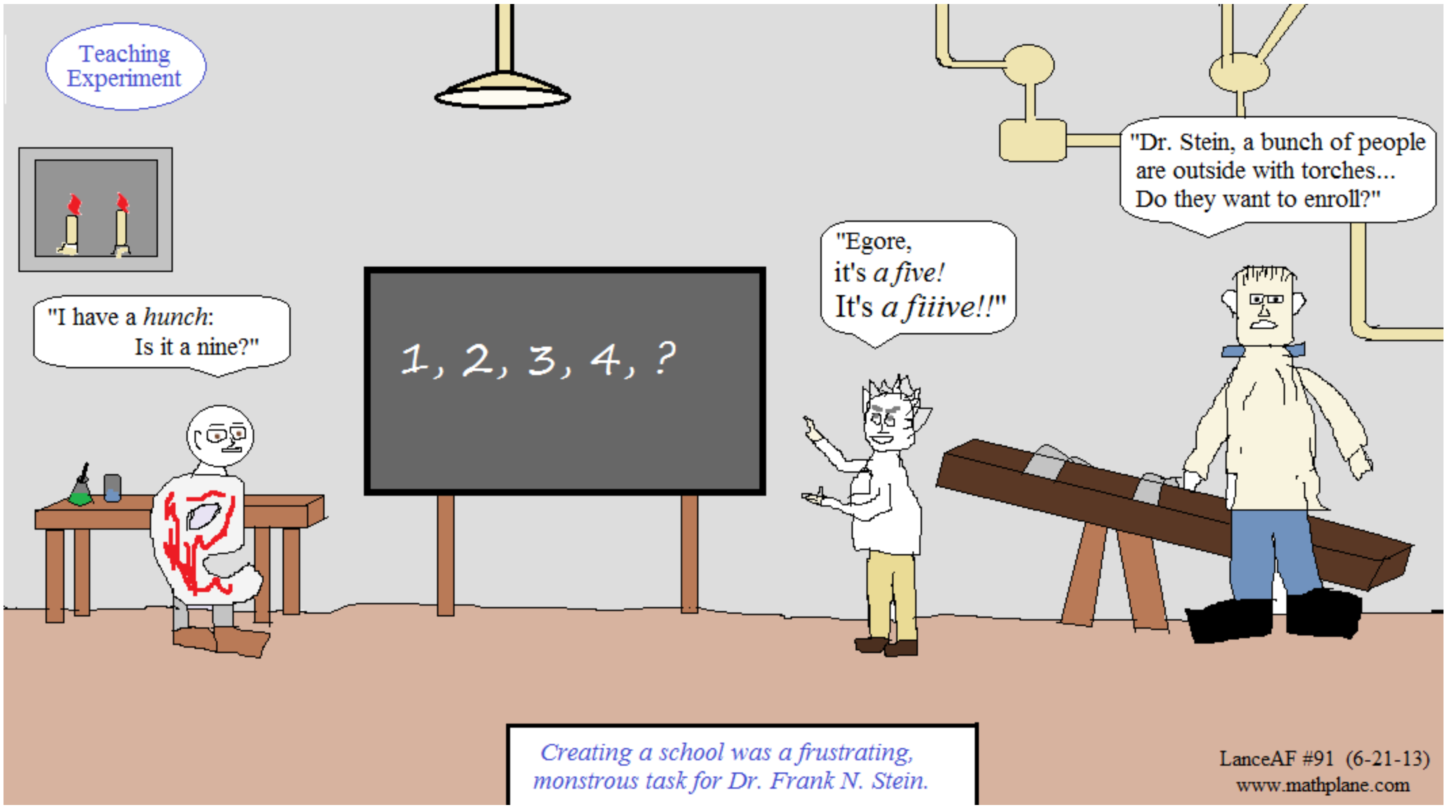
Check: $n = 21$

$a_1 = 15$ $a_{21} = 15 + 3(21 - 1) = 75$ seats

Total seats with 21 rows: $S_{21} = \frac{21(15 + 75)}{2}$

$= 945$

If there were 20 rows, the theater would
only have 945 - 75 = 870 seats...



Creating a school was a frustrating, monstrous task for Dr. Frank N. Stein.

Another Practice Test -→

Sequences & Series Test II

Part I: Sequences, Series, and Terms

1) Questions

   a) What is the arithmetic mean between 10 and 20?

   b) What is the geometric mean between 10 and 20?

2) Write the first 3 terms:

   a) arithmetic series; 1st term: 50
   common difference: 1.6

   b) geometric sequence; 1st term: 13
   common ratio: 4

3) In a geometric sequence, $T_3 = 24 \qquad T_5 = 6$

   a) What is T_4 ?

   b) What is another possible answer?

Part II: Sums

Find sum: 1) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \ldots$

2) $1.5 + 4.5 + 13.5 + \ldots$

Partial sum:
(sigma notation)

Sequences & Series Test II

3) $\sum_{k=1}^{11} 15\left(\frac{2}{3}\right)^{k} =$

4) $\sum_{k=0}^{20} 100\,(1.08)^{k} =$

5) Find the sum of *multiples of 4* between 15 and 523.

6) Find the sum of all *multiples of 7* including and between 7 and 98.

7) If the sum of the first 14 terms of arithmetic sequence is 301, and the first term is 2, what is the 4th term?

Sequences & Series Test II

Part III: Word Problems & Applications

1) A tree grows geometrically.
The first year growth was 40 inches.
The second year growth was 38 inches.

a) Predict how much the tree will growth in the 5th year

b) How tall will the tree be after 10 years?

c) What is the ultimate height of the tree?

2) A ball is dropped 500ft and it bounces 60% of the distance of the previous fall...
How high does the ball bounce after the 7th bounce?
How far does a ball travel when it hits the ground for the 10th time?

3) A parent adds 1000 dollars per year into the kid's college fund that pays 3.2% annual compound interest.
How much money will be in the account after 18 years?

Sequences & Series Test II

Part IV: Miscellaneous

1) Find the 1st five terms and S_5 of the sequence $a_n = \frac{1}{2^n} \log 1000^n$

2) $a_n = \ln(1 \cdot 2 \cdot 3 \cdot .. \cdot n)$

Find S_4

3) $t_1 = -40$ $r = -3/8$

Does this series converge? If so, what value?

4) $-90 + 81 + -72.9 + \ldots$

Does this series converge? If so, what value?

5) Write $\frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4}$ using factorials

$\frac{12!}{10!4!} =$

$\frac{(n+1)!}{(n-1)!} =$

SOLUTIONS -→

Sequences & Series Test II

SOLUTIONS

Part I: Sequences, Series, and Terms

1) Questions

a) What is the arithmetic mean between 10 and 20? 15

b) What is the geometric mean between 10 and 20? $10\sqrt{2}$

2) Write the first 3 terms:

a) arithmetic series; 1st term: 50
common difference: 1.6

$50 + 51.6 + 53.2$

series is the sum of terms

b) geometric sequence; 1st term: 13
common ratio: 4

13, 52, 208

13 x 4 = 52

52 x 4 = 208

3) In a geometric sequence, $T_3 = 24$ $T_5 = 6$

a) What is T_4 ? 12

b) What is another possible answer? -12

"geometric mean"

6 x 24 = 144

since there is one term in between, square root:

$$\sqrt{144} = 12$$

also,

$$\frac{6}{24} = \frac{1}{4} \qquad \sqrt{\frac{1}{4}} = \frac{1}{2}$$

common ratio

$$24 \underbrace{\qquad}_{\text{x } 1/2} 12 \underbrace{\qquad}_{\text{x } 1/2} 6$$

Part II: Sums

Find sum: 1) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \ldots$

common ratio: $\frac{-3}{6} = \frac{3/2}{-3} = \frac{-3/4}{3/2} = \frac{-1}{2}$

$$S_\infty = \frac{6}{1-(-1/2)} = \boxed{4}$$

$$\boxed{S_\infty = \frac{a_1}{1-r}}$$

2) $1.5 + 4.5 + 13.5 + \ldots$

since the common ratio is 3 > 0, the series *diverges* (i.e. goes to infinity)

Partial sum:
(sigma notation)

SOLUTIONS

3) $\sum_{k=1}^{11} 15\left(\frac{2}{3}\right)^k =$

$a_1 = 10$

$r = \frac{2}{3}$

$$\frac{10\left(1-\left(\frac{2}{3}\right)^{11}\right)}{1-\frac{2}{3}} \approx \frac{10(.9884)}{1/3}$$

$10,\ 6.67,\ 4.44,\ 2.96,\ 1.97,\ \dots \longrightarrow \boxed{\approx\ 29.65}$

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

4) $\sum_{k=0}^{20} 100\,(1.08)^k =$

Important: There are 21 terms! (not 20)

$r = 1.08$

$a_0 = 100$

$a_1 = 108$

$$S_{21} = \frac{100(1-(1.08)^{21})}{1-1.08} \quad \boxed{= 5042.29}$$

$$\text{or,}\quad 100 + \frac{108(1-(1.08)^{20})}{1-1.08}$$

5) Find the sum of *multiples of 4* between 15 and 523.

First, determine the sequence (i.e. 1st and last terms)... $a_1 = 16$ and $a_k = 520$

Each multiple will be 4k

$$\frac{520-16}{4} = 126 \text{ 'moves' from 1st to last term}$$

$$\sum_{k=1}^{127} 4k+12 = \frac{127}{2}(16+520) = \boxed{34{,}036}$$

6) Find the sum of all *multiples of 7* including and between 7 and 98.

There are 14 multiples of 7 between 7 and 98...

105

$$\sum_{k=1}^{14} 7k = 7 + 14 + 21 + \dots + 91 + 98$$

105

7 pairs of 105...

$(105) \times 7 = \boxed{735}$

7) If the sum of the first 14 terms of arithmetic sequence is 301, and the first term is 2, what is the 4th term?

Step 1: Find the last term

$a_1 = 2$

14 terms

$$\frac{14\,(2+a_{14})}{2} = 301$$

$$2 + a_{14} = 43$$

$$a_{14} = 41$$

Step 2: Find the common difference

$$d = \frac{41-2}{13} = 3$$

Step 3: Find the 4th term... $2,\ 5,\ 8,\ \boxed{11,}\ 14\ 17..$

Sequences & Series Test II

SOLUTIONS

Part III: Word Problems & Applications

$$T_n = T_1(r^{n-1})$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

1) A tree grows geometrically.
The first year growth was 40 inches.
The second year growth was 38 inches.

a) Predict how much the tree will growth in the 5th year $40(.95)^4 = \boxed{32.58 \text{ inches}}$

b) How tall will the tree be after 10 years? $\boxed{321.01 \text{ inches}}$

$$S_{10} = \frac{40\,(1 - (.95)^{10})}{1 - .95} = 321.01$$

c) What is the ultimate height of the tree? $\boxed{800 \text{ inches}}$

$$\frac{40}{1 - (.95)} = 800$$

$$S_\infty = \frac{a_1}{1 - r}$$

a_n = growth in nth year (inches)

$a_1 = 40$

$a_2 = 38$

common ratio (r) = $\frac{38}{40}$ .95

2) A ball is dropped 500ft and it bounces 60% of the distance of the previous fall...
How high does the ball bounce after the 7th bounce? $300(.6)^{n-1} = \boxed{14}$ (note: the 1st bounce is 300 ft!)
How far does a ball travel when it hits the ground for the 10th time?

find travel total (up)

$$\frac{300(1 - .6^9)}{(1 - .6)}$$

742.44

find travel total (down)

$$\frac{500(1 - .6^{10})}{(1 - .6)}$$

1242.44

$\boxed{\text{total: } 1984.88}$

3) A parent adds 1000 dollars per year into the kid's college fund that pays 3.2% annual compound interest.
How much money will be in the account after 18 years?

Step 1: Find the sum formula (sigma notation)

Step 2: Find the first 2 terms a_1 and a_2 to determine the ratio (r)

Step 3: Insert into the partial sum formula

$$\sum_{k=1}^{18} \underbrace{1000}_{\text{deposit}}\underbrace{(1 + .032)^{19 - k}}_{\text{years THAT deposit compounds}}$$

$a_1 = 1000(1.032)^{18} = 1762.93$

$a_2 = 1000(1.032)^{17} = 1708.26$

$$r = \frac{1708.26}{1762.93} \approx .97$$

$$S_{18} = \frac{1762.93\,(1 - .97^{18})}{(1 - .97)} = 24{,}801.53 \text{ Approximate}$$

mathplane.com

Sequences & Series Test II

SOLUTIONS

Part IV: Miscellaneous

1) Find the 1st five terms and S_5 of the sequence $a_n = \frac{1}{2^n} \log 1000^n$

First five terms: $\frac{3}{2}$ $\frac{3}{2}$ $\frac{9}{8}$ $\frac{3}{4}$ $\frac{15}{32}$

$S_5 = \frac{171}{32}$ (add up the 5 terms)

2) $a_n = \ln(1 \cdot 2 \cdot 3 \cdot \ldots \cdot n)$

Find S_4

$a_1 = \ln(1)$

$a_2 = \ln(2)$

$a_3 = \ln(6)$

$a_4 = \ln(24)$

$S_4 = 0 + .693 + 1.792 + 3.178$

≈ 5.663

3) $t_1 = -40$ $r = -3/8$

Does this series converge? If so, what value?

since the $| -3/8 | < 1$, it does converge

$\frac{-40}{1 - (-3/8)}$ -29.09

4) $-90 + 81 + -72.9 + \ldots$

Does this series converge? If so, what value?

since the common ratio is -.9, it converges

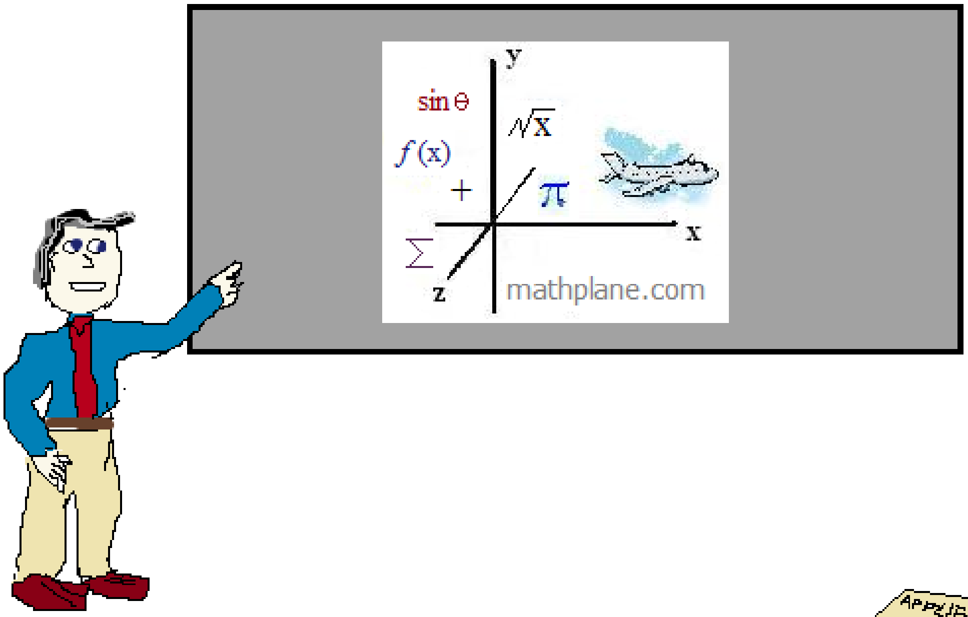
$| -.9 | < 1$

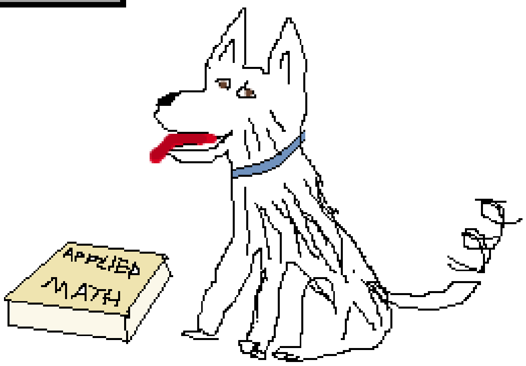
$\frac{-90}{1 - (-.9)}$ -47.368

5) Write $\frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4}$ using factorials $\frac{9!}{6!\ 4!}$

$$\frac{12!}{10!4!} = \frac{12 \cdot 11 \cdot 10!}{10! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{132}{24} = 5.5$$

$$\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)(n-1)(n-2)\ldots}{(n-1)(n-2)\ldots} = n^2 + n$$





Also, at mathplane.ORG for mobile and tablets.

One more puzzle-→

Hidden Message

Hint: Ordered lines of prisoners?

Letter/Number Key

1	2	3	4	5	6	7	8	9	0
C	E	G	N	O	R	Q	S	T	U

Answer the 12 questions...
Then, convert the numbers into letters to reveal the answer!

---

1) In the arithmetic sequence, $T_3 = 24$ and $T_8 = 49$. What is T_{22}?

1 ☐ 9 → ____

2) What is the geometric mean between 5 and 125?

2 ☐ → ____

3) What is the next term in the following sequence?

64, -32, 16, -8, ?

☐ → ____

4) In the following arithmetic sequence, 48 is what term?

99, 96, 93, ...

1 ☐ → ____

5) What is the common ratio of the following sequence?

$$\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$$

☐ → ____

6) What is the limit to which this geometric series converges?

38 + 19 + 9.5 + ...

☐ 6 → ____

7) $\sum_{n=3}^{7} n^2 - 3n$

6 ☐ → ____

8) What is the 20th partial sum of the following series?

First term: -2 Common difference: 4

7 ☐ 0 → ____

9) In the Fibonacci sequence 2, 3, 5, 8, 13, 21,.. what term is next?

3 ☐ → ____

10) What is the sum of the first 9 terms of the geometric series?

first term: 6561 common ratio: 1/3

984 ☐ → ____

11) What is the arithmetic mean between -6 and 10?

☐ → ____

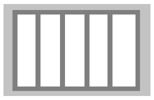
12) In the following (recursive) sequence, identify the *5th* term:

$$a_1 = 3$$
$$a_{n+1} = 3a_n + 1$$

2 ☐ 3 → ____

Hidden Message

SOLUTIONS



Hint: Ordered lines of prisoners?

Letter/Number Key

1	2	3	4	5	6	7	8	9	0
C	E	G	N	O	R	Q	S	T	U

Answer the 12 questions...
Then, convert the numbers into letters
to reveal the answer!

---

1) In the arithmetic sequence, $T_3 = 24$ and $T_8 = 49$. What is T_{22}?

$a_n = a_1 + (n - 1)d$

$24 = T_1 + (3 - 1)(5)$

$T_1 = 14$

common difference

$d = \frac{49 - 24}{5} = 5$

$T_{22} = 14 + (22 - 1)(5) = \boxed{119}$

$1\boxed{1}9 \rightarrow$ C

2) What is the geometric mean between 5 and 125?

$5 \cdot 125 = 625$ then, $\sqrt{625} = \boxed{25}$

note: $\sqrt{\frac{125}{5}} = 5$ so common ratio is 5

$2\boxed{5} \rightarrow$ O

3) What is the next term in the following sequence?

64, -32, 16, -8, ?

(common ratio is -1/2)

16, -8, $\boxed{4,}$ -2, 1....

$\boxed{4} \rightarrow$ N

4) In the following arithmetic sequence, 48 is what term?

common difference d = -3

99, 96, 93, ...

$T_3 = 93$

48 - 93 = -45

93 to 48 is

$\frac{-45}{-3} = 15$ "moves"

$\boxed{\text{18th term..}}$

$1\boxed{8} \rightarrow$ S

5) What is the common ratio of the following sequence?

$\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$

$\frac{1/6}{1/12} = 2 \quad \frac{2/3}{1/3} = \boxed{2}$

$\boxed{2} \rightarrow$ E

6) What is the limit to which this geometric series converges?

common ratio $r = \frac{1}{2}$

38 + 19 + 9.5 + ...

$S_\infty = \frac{T_1}{1 - r} = \frac{38}{1/2} = \boxed{76}$

$\boxed{7}6 \rightarrow$ Q

7) $\sum_{n=3}^{7} n^2 - 3n$

3: (9 - 9) = 0 4: (16 - 12) = 4 5: (25 - 15) = 10 6: (36 - 18) = 18 7: (49 - 21) = 28

$0 + 4 + 10 + 18 + 28 = \boxed{60}$

$6\boxed{0} \rightarrow$ U

8) What is the 20th partial sum of the following series?

First term: -2 Common difference: 4

first, find the 20th term: $T_{20} = -2 + (20 - 1)(4) = 74$

$\overbrace{-2 + \underbrace{2 + \overbrace{6 + \quad\quad + 66}^{72} + 70}_{72} + 74}$

72

72 x (10 pairs) = $\boxed{720}$

$7\boxed{2}0 \rightarrow$ E

9) In the Fibonacci sequence 2, 3, 5, 8, 13, 21,.. what term is next?

2 + 3 = 5... 3 + 5 = 8... 8 + 13 = 21... etc...

$\boxed{13 + 21 = 34}$

$3\boxed{4} \rightarrow$ N

10) What is the sum of the first 9 terms of the geometric series?

first term: 6561 common ratio: 1/3

$S_n = (a_1)\frac{1 - r^n}{1 - r}$

$S_9 = (6561)\frac{1 - (1/3)^9}{1 - (1/3)} = \boxed{9841}$

$984\boxed{1} \rightarrow$ C

11) What is the arithmetic mean between -6 and 10?

$\frac{-6 + 10}{2} = \boxed{2}$

$\underbrace{-6,\ 2}_{+8}\underbrace{\ ,\ 10}_{+8}$

$\boxed{2} \rightarrow$ E

12) In the following (recursive) sequence, identify the *5th* term:

$a_1 = 3$

$a_{n+1} = 3a_n + 1$

$a_1 = 3$

$a_2 = 3(3) + 1 = 10$

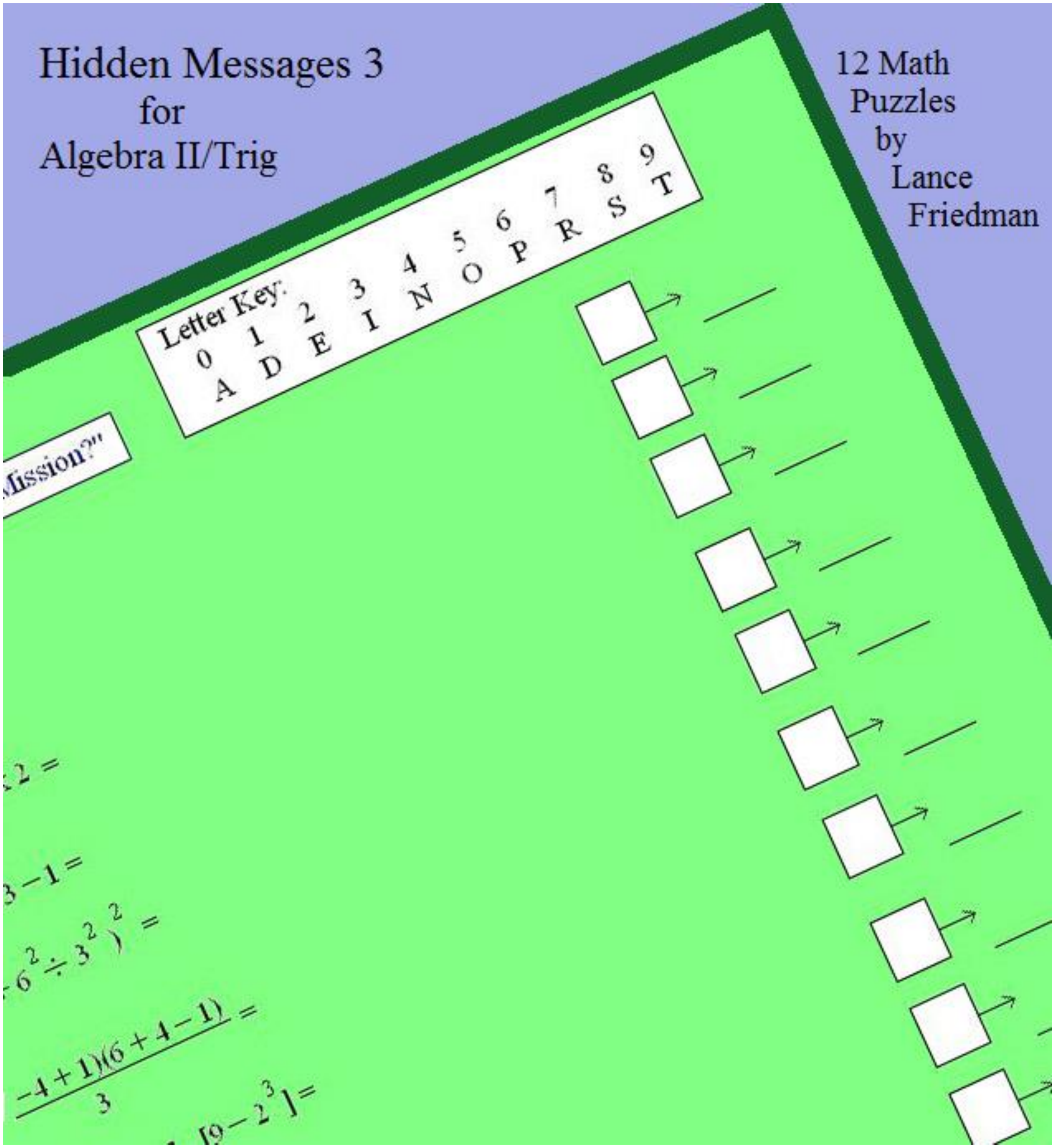
$a_3 = 3(10) + 1 = 31$

$a_4 = 3(31) + 1 = 94$

$a_5 = 3(94) + 1 = \boxed{283}$

$2\boxed{8}3 \rightarrow$ S

"CON" Sequences



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